

Brewster cross-polarization

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We theoretically derive the polarization-resolved intensity distribution of a TM -polarized fundamental Gaussian beam reflected by an air-glass plane interface at Brewster incidence. The reflected beam has both a dominant (TM) and a cross-polarized (TE) component, carried by a TEM_{10} and a TEM_{01} Hermite-Gaussian spatial mode, respectively. Remarkably, we find that the TE -mode power scales quadratically with the angular spread of the incident beam and it is comparable to the TM -mode power. Experimental confirmations of the theoretical results are also presented. © 2009 Optical Society of America

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When a beam of light impinges upon a plane interface separating two transparent media, it produces reflected and transmitted beams. In 1815 the Scottish physicist David Brewster discovered the total polarization of the reflected beam at the angle θ_B since named after him [1]. From his observations he was also able to empirically determine the celebrated equation, known as Brewster's law, $\tan \theta_B = n_1/n_2$, where n_1 and n_2 are the respective refractive indices of the two media. Several articles have been published on theory and experiments about Brewster's law for beams with non-planar wave fronts. Fainman and Shamir [2] addressed reflection of an (isotropic) spherical wave, by a Brewster angle polarizer; they found a cross-polarized component. More recently, Kőházi-Kis [3] has theoretically derived and experimentally confirmed cross-polarization effects occurring at Brewster incidence. Shortly afterwards, Li and Vernon [4] addressed the same problem, using a microwave Gaussian beam; they do not mention cross polarization. Consequences of cross-polarization coupling (XPC) at dielectric interfaces were also theoretically investigated by Nasalski and coworkers in a series of interesting papers [5, 6].

However, these studies fail to compare the intrinsic XPC due to the natural angular spread (namely, the focusing) of the incident beam, and the non-intrinsic one caused by reflection of such a beam at a dielectric interface. The main aim of this Letter is to fill this gap.

The structure of this Letter is as follows. We first solve the general problem of the reflection of a polarized fundamental Gaussian beam at the plane interface between two optical media [7, 8]. Next, we derive analytical expressions for the polarization-dependent transverse spatial profiles of the reflected beam. From this result, we are able to prove that non-intrinsic

XPC scales quadratically with the angular spread θ_0 of the incident beam, as opposed to the intrinsic XPC that scales as the fourth power of θ_0 . Finally, we present experimental confirmations of our theoretical findings.

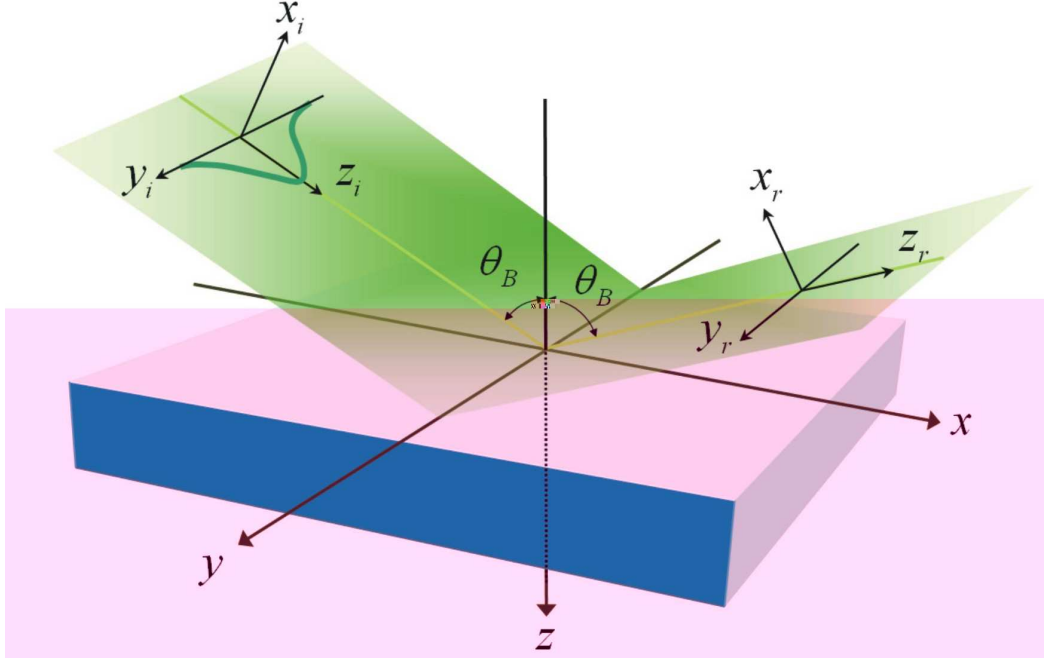


Fig. 1. (Color online) Geometry of beam reflection at the air-medium interface. θ_B is the Brewster angle.

Consider a monochromatic beam of light incident upon a plane interface that separates air from glass. With $n = n_{\text{air}}/n_{\text{glass}}$ we denote the ratio between the two refractive indices. As the beam meets the interface coming from the air side, it will be convenient to take the axis z of the laboratory Cartesian frame $K = (O, x, y, z)$ normal to the interface and directed from the air to the glass. Moreover, we choose the origin O in a manner that the plane interface has equation $z = 0$. The air-glass interface, the incident and the reflected beams

are pictorially illustrated in Fig. 1. In addition to the laboratory frame, we use a Cartesian frame $K_i = (O, x_i, y_i, z_i)$ attached to the incident beam and another one $K_r = (O, x_r, y_r, z_r)$ attached to the reflected beam. Let $\mathbf{k}_0 = k_0 \hat{\mathbf{z}}_i$ and \mathbf{k} denote the central and noncentral wave vectors of the incident beam, respectively, with $|\mathbf{k}| = |\mathbf{k}_0| = k_0$. Then, the electric field of the incident beam can be written as a linear superposition of the fundamental vector plane-wave mode functions $\hat{\chi}_\lambda(\mathbf{k}) = \hat{\mathbf{e}}_\lambda(\mathbf{k}) \exp(i\mathbf{k} \cdot \mathbf{r})$ with complex amplitudes $a_\lambda(\mathbf{k})$, as follows:

$$\mathbf{E}^I(\mathbf{r}) = \sum_{\lambda=1}^2 \int a_\lambda(\mathbf{k}) \hat{\chi}_\lambda(\mathbf{k}) d^2 k_T, \quad (1)$$

where $\mathbf{k}_T = \mathbf{k} - \mathbf{k}_0(\mathbf{k}_0 \cdot \mathbf{k})/k_0^2$ is the transverse part of \mathbf{k} , and we choose the polarization unit basis vectors as $\hat{\mathbf{e}}_1(\mathbf{k}) = \hat{\mathbf{e}}_2(\mathbf{k}) \times \mathbf{k}/k_0$, and $\hat{\mathbf{e}}_2(\mathbf{k}) = \hat{\mathbf{z}} \times \mathbf{k}/|\hat{\mathbf{z}} \times \mathbf{k}|$ [9]. Here $\hat{\mathbf{z}}$ is a real unit vector directed along the laboratory axis z . By definition, $\hat{\mathbf{e}}_1(\mathbf{k})$ lies in the plane of incidence containing both the wave vector \mathbf{k} and $\hat{\mathbf{z}}$, while $\hat{\mathbf{e}}_2(\mathbf{k})$ is orthogonal to such a plane. A plane wave whose electric field vector is parallel to either $\hat{\mathbf{e}}_1(\mathbf{k})$ or $\hat{\mathbf{e}}_2(\mathbf{k})$, is referred to as either a *TM* or a *TE* wave, respectively. The symbols *S* for *TE* and *P* for *TM*, are also widely used. In Eq. (1) $a_\lambda(\mathbf{k}) = A(\mathbf{k}) \alpha_\lambda(\mathbf{k})$, where $A(\mathbf{k})$ and $\alpha_\lambda(\mathbf{k})$ are the scalar and the vector spectral amplitudes of the field, respectively. Here we consider a monochromatic Gaussian beam, whose spectral amplitude $A(\mathbf{k})$ is localized in \mathbf{k} space, centered at the central wave vector $\mathbf{k}_0 = k_0 \hat{\mathbf{z}}_i$, on the sphere of equation $\omega^2(\mathbf{k}) = c^2 k_0^2$, namely

$$A(\mathbf{k}) = e^{-\frac{|\mathbf{k}_T/k_0|^2}{\theta_0^2}} e^{ik_0 d(1-|\mathbf{k}_T/k_0|^2)^{1/2}}, \quad (2)$$

where $\theta_0 \equiv 2/(k_0 w_0)$ is the diffraction-defined angular aperture of the incident beam [10] which has, by hypothesis, a minimum diameter (spot size) equal to $2w_0$ located at $z_i = -d$.

The vector spectral amplitudes are defined as $\alpha_\lambda(\mathbf{k}) = \hat{\mathbf{e}}_\lambda(\mathbf{k}) \cdot \hat{\mathbf{f}}$, where $\hat{\mathbf{f}} = (f_P \hat{\mathbf{x}}_i + f_S \hat{\mathbf{y}}_i)$, with $|f_P|^2 + |f_S|^2 = 1$, is a complex-valued unit vector that fixes the polarization of the incident beam.

When the latter is reflected at the interface, each vector mode function changes according to

$$\hat{\chi}_\lambda(\mathbf{k}) \mapsto r_\lambda(\mathbf{k}) \hat{\chi}_\lambda(\tilde{\mathbf{k}}), \quad (3)$$

where $r_1(\mathbf{k})$ and $r_2(\mathbf{k})$ are the Fresnel reflection amplitudes for *TM* and *TE* waves, respectively [11], and $\tilde{\mathbf{k}} = \mathbf{k} - 2\hat{\mathbf{z}}(\hat{\mathbf{z}} \cdot \mathbf{k})$ is sets by the law of specular reflection [12]. If we substitute Eq. (3) into Eq. (1), we obtain

$$\mathbf{E}^I(\mathbf{r}) \mapsto \mathbf{E}^R(\mathbf{r}) = \sum_{\lambda=1}^2 \int a_\lambda(\mathbf{k}) r_\lambda(\mathbf{k}) \hat{\chi}_\lambda(\tilde{\mathbf{k}}) d^2 k_T, \quad (4)$$

where $\tilde{\mathbf{k}}_0 = k_0 \hat{\mathbf{z}}_r$, by definition. The expression for the magnetic field $\mathbf{B}^R(\mathbf{r})$ of the reflected beam may be obtained from the equation above via the straightforward substitutions $a_\lambda(\mathbf{k}) \rightarrow b_\lambda(\mathbf{k})/c$, where $b_1(\mathbf{k}) = -a_2(\mathbf{k})r_2(\mathbf{k})/r_1(\mathbf{k})$, and $b_2(\mathbf{k}) = a_1(\mathbf{k})r_1(\mathbf{k})/r_2(\mathbf{k})$.

From Eq. (2) it follows that $A(\mathbf{k}) \simeq 0$ for those wave vectors \mathbf{k} lying outside the paraxial domain $\mathcal{P} = \{\mathbf{k} : |\mathbf{k}_T/k_0| \lesssim \theta_0\}$, with $\theta_0 \ll 1$ for well-collimated beams. This allows us to calculate analytically \mathbf{E}^R (and, similarly, \mathbf{B}^R) via a power series expansion for the integrand of Eq. (4) about the point $\mathbf{k} = \mathbf{k}_0$ up to and including second order terms in \mathbf{k}_T/k_0 . In practice, we extend to the problem at hand, the perturbative approach introduced by Lax *et al.*, [13], and further developed by Deutsch and Garrison [14]. It is easy to see that if with $\gamma(\mathbf{k}) = \arccos(\mathbf{k} \cdot \mathbf{k}_0/k_0^2)$ we denote the angle between the central wave vector \mathbf{k}_0 and the

non-central one \mathbf{k} , then we can write $|\mathbf{k}_T/k_0| = \sin \gamma(\mathbf{k}) \simeq \gamma(\mathbf{k})$, where $\gamma(\mathbf{k}) \lesssim \theta_0 \ll 1$, being θ_0 the natural small parameter for the power series expansion [14]. Explicit expressions for the power series expansions of both $\mathbf{E}^R(\mathbf{r})$ and $\mathbf{B}^R(\mathbf{r})$ are given in Appendix A.

From the knowledge of both $\mathbf{E}^R(\mathbf{r})$ and $\mathbf{B}^R(\mathbf{r})$ it is possible to calculate the intensity spatial distribution (i.e., the beam profile) $I(\mathbf{r})$ of the reflected beam, as the flux of the cycle-averaged Poynting vector $\bar{\mathbf{S}} \propto \text{Re}(\mathbf{E}^R \times \mathbf{B}^{R*})$ across a surface perpendicular to the central direction of propagation $\hat{\mathbf{z}}_r$, namely $I(\mathbf{r}) \propto \bar{\mathbf{S}} \cdot \hat{\mathbf{z}}_r$. Since $(\mathbf{E}^R \times \mathbf{B}^{R*}) \cdot \hat{\mathbf{z}}_r = E_{x_r}^R B_{y_r}^{R*} - E_{y_r}^R B_{x_r}^{R*}$, we can write $I(\mathbf{r}) = I_P(\mathbf{r}) + I_S(\mathbf{r})$, where $I_P(\mathbf{r}) = \text{Re}(E_{x_r}^R B_{y_r}^{R*})$, and $I_S(\mathbf{r}) = \text{Re}(-E_{y_r}^R B_{x_r}^{R*})$ are the intensity distributions produced by the P - and S -polarized components of the reflected beam, respectively. Here P and S polarization directions are defined with respect to the central plane of incidence containing $\hat{\mathbf{z}}$, \mathbf{k}_0 , and $\tilde{\mathbf{k}}_0$. After a lengthy but straightforward calculation it is not difficult to obtain, for a P -polarized incident beam (i.e., for the choice $f_P = 1$ and $f_S = 0$), the following power series expansions:

$$I_P(\mathbf{r})/I_0(\mathbf{r}) = r_P^2 + \theta_0 u X + \theta_0^2 (v + pX^2 + qY^2), \quad (5)$$

$$I_S(\mathbf{r})/I_0(\mathbf{r}) = \theta_0^2 sY^2, \quad (6)$$

where $X = x_r/w_0$, $Y = y_r/w_0$, $Z = (z_r + d)/L$, and $I_0(\mathbf{r}) = \exp\left(-2\frac{X^2+Y^2}{1+Z^2}\right)/(1+Z^2)$ is the intensity distribution of the incident beam. In Eq. (5) the transverse coordinates x_r and y_r are normalized with respect to the beam waist w_0 , while the longitudinal coordinate z_r is normalized with respect to the Raleigh range $L = kw_0^2/2$ of the beam. In Eqs. (5) and (6), r_P and r_S are the Fresnel reflection amplitude for P and S waves, respectively, evaluated at the

central angle of incidence $\theta = \arccos(\mathbf{k}_0 \cdot \hat{\mathbf{z}}/k_0)$, while u , v , p , q and s are some complicated functions of Z, θ, r_P, r_S and their derivatives [15] whose explicit form is given in Appendix B.

However, at the Brewster angle of incidence $\theta = \theta_B \equiv \arctan(n)$, only p and s take a non-zero value, namely

$$p(1 + Z^2) = (\partial r_P / \partial \theta)|_{\theta_B}^2, \quad s(1 + Z^2) = r_S^2 / n^2|_{\theta_B}, \quad (7)$$

and Eqs. (5) and (6) reduces to

$$I_P(\mathbf{r})/I_0(\mathbf{r}) = \theta_0^2 p X^2, \quad I_S(\mathbf{r})/I_0(\mathbf{r}) = \theta_0^2 s Y^2, \quad (8)$$

respectively. From this result it immediately follows that the ratio ρ between the power of the S and the P components of the reflected beam at Brewster incidence, is simply equal to s/p ,

$$\rho = \frac{\iint I_S(\mathbf{r}) dX dY}{\iint I_P(\mathbf{r}) dX dY} = \left(\frac{r_S}{n} \frac{1}{\partial r_P / \partial \theta} \right)_{\theta=\theta_B}^2. \quad (9)$$

This simple result is remarkable since it shows that ρ is *independent* from the waist w_0 of the incident beam, that is, ρ take the same value for either a well-collimated or a strongly-focussed beam.

Equations (5-6) represent the main theoretical result of this Letter. In particular, Eq. (6) shows that non-intrinsic XPC generates an intensity $I_S(\mathbf{r})$ that scales quadratically with θ_0 . This behavior cannot be ascribed to the intrinsic XPC exhibited by the incident beam since the latter is due to the beam-divergence only and the consequent cross-polarized intensity

scales with θ_0^4 [16, 17]. Moreover, Eq. (8) shows two important things. First, we see that even at Brewster angle of incidence the extinction of a P -polarized beam is not perfect, as $(dr_P/d\theta)|_{\theta_B} \neq 0$ and $r_S|_{\theta_B} \neq 0$. Thus, although the input beam is P -polarized, a S component appears after reflection. Second, the two expressions in Eq. (8) show that after reflection the cylindrical symmetry about the axis of propagation of the beam is lost and two orthogonally polarized TEM_{10} and TEM_{01} modes are generated. The fact that I_S has a TEM_{01} profile, as opposed to the cloverleaf TEM_{11} pattern typical of the cross polarization intensity of the incident beam, is consistent with the hypothesis that such a term originates from non-intrinsic XPC and it is not a simple beam-divergence effect.

We verified these theoretical results in our laboratory by using a Super-luminescent Light Emitting Diode (SLED) operating at $\lambda = 820 \text{ nm}$ (InPhenix IPSDD0802) as a light source. The output of the SLED was first spatially filtered by a single-mode optical fiber to prepare the input beam into the fundamental Gaussian mode, and then collimated by a microscope objective to produce a very large beam waist, ($w_0 = 1.64 \text{ mm}$) before passing across a polarizer selecting P -polarization. A second lens put behind the polarizer generated the desired waist for the input beam. The so-prepared beam was sent upon the surface of a right-angle BK7 glass ($n = 1.51$) prism mounted on a precision rotation stage with a resolution of $9 \times 10^{-6} \text{ rad}$ (Newport URS-BCC) to accurately determine the Brewster angle ($\theta_B \cong 56.49^\circ$). Finally, the polarization-dependent beam intensity profiles after reflection were recorded by a CCD-based Beam Intensity Profiler (Spiricon LBA-FW-SCOR-20) mounted behind a polarizer put along the axis of the reflected beam, at large distance from the interface (far

field measurement).

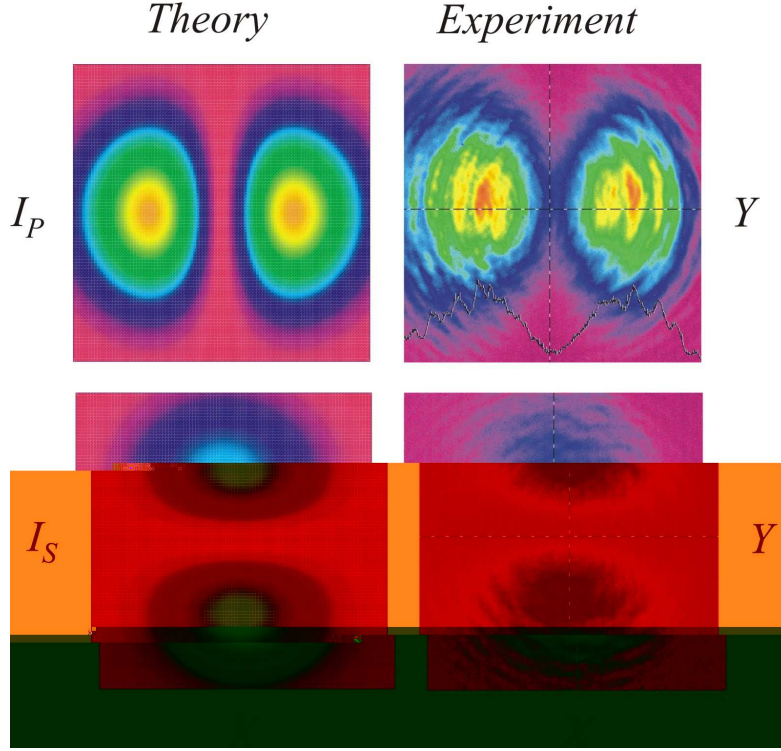


Fig. 2. (Color online) Calculated and measured intensity transverse spatial profiles of the P - and S -polarized modes of the reflected beam. The beam waist of the incident beam was $w_0 = 34 \mu\text{m}$.

A qualitative comparison between calculated and measured intensity distributions is shown in Fig. 2. The *measured* ratio ρ_{exp} of the S -polarization component power to the P -polarization one at Brewster incidence, was $\rho_{\text{exp}} = 0.20 \pm 0.05$ in excellent agreement with the theoretical prediction of Eq. (9) giving, for BK7 glass, $\rho_{\text{th}} = 4n^4/(1+n^2)^4 \cong 0.18$.

In conclusion, we found that when a TM -polarized fundamental Gaussian beam is reflected at Brewster incidence it generates a two-mode beam with both a dominant and a cross-polarized component. The intensity of the latter scales quadratically with the angular divergence of the incident beam and can be, therefore, orders of magnitude bigger than the intrinsic cross-polarized intensity of the incident beam that scales with the fourth power of θ_0 .

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Appendix A

As a result of the power series expansion truncated at second order terms in θ_0 , we found the following expressions for the electric and magnetic fields for the reflected beam, evaluated in the beam frame K_r :

$$\mathbf{E}^R(\mathbf{r}) = \frac{\exp\left(-i\frac{X^2+Y^2}{i-Z}\right)}{i-Z} (\hat{\mathbf{x}}_r E_{x_r} + \hat{\mathbf{y}}_r E_{y_r} + \hat{\mathbf{z}}_r E_{z_r}), \quad (10)$$

$$\mathbf{B}^R(\mathbf{r}) = \frac{\exp\left(-i\frac{X^2+Y^2}{i-Z}\right)}{i-Z} (\hat{\mathbf{x}}_r B_{x_r} + \hat{\mathbf{y}}_r B_{y_r} + \hat{\mathbf{z}}_r B_{z_r}), \quad (11)$$

where $X = x_r/w_0$, $Y = y_r/w_0$, $Z = (z_r + d)/L$, and

$$E_{x_r} = r_P + \theta_0 X \frac{r'_P}{i - Z} + \frac{\theta_0^2}{2} \left\{ i \frac{(r''_P - 2r_P)}{2(i - Z)} + i \frac{r'_P \cot \theta}{2(i - Z)} - \frac{i(r_P + r_S) \cot^2 \theta}{i - Z} \right. \\ \left. + X^2 \frac{(r''_P - 2r_P)}{(i - Z)^2} + Y^2 \left[\frac{r'_P \cot \theta}{(i - Z)^2} - \frac{2(r_P + r_S) \cot^2 \theta}{(i - Z)^2} \right] \right\}, \quad (12)$$

$$E_{y_r} = \theta_0 Y \frac{(r_P + r_S) \cot \theta}{i - Z} + \theta_0^2 XY \left[\frac{(r'_P + r'_S) \cot \theta}{(i - Z)^2} + \frac{r_S - (r_P + r_S) \csc^2 \theta}{(i - Z)^2} \right], \quad (13)$$

$$E_{z_r} = \theta_0 \frac{r_P X}{i - Z} + \theta_0^2 \left[i \frac{r'_P}{2(i - Z)} + i \frac{(r_P + r_S) \cot \theta}{2(i - Z)} + X^2 \frac{r'_P}{(i - Z)^2} + Y^2 \frac{(r_P + r_S) \cot \theta}{(i - Z)^2} \right], \quad (14)$$

and

$$B_{x_r} = -\theta_0 Y \frac{(r_P + r_S) \cot \theta}{i - Z} + \theta_0^2 XY \left[-\frac{(r'_P + r'_S) \cot \theta}{(i - Z)^2} + \frac{(r_P + r_S) \cot^2 \theta}{(i - Z)^2} \right], \quad (15)$$

$$B_{y_r} = r_P + \theta_0 X \frac{r'_P}{i - Z} + \frac{\theta_0^2}{2} \left\{ i \frac{r'_P \cot \theta}{2(i - Z)} + i \frac{r''_P + 2r_S - 2(r_P + r_S) \csc^2 \theta}{2(i - Z)} \right. \\ \left. + X^2 \frac{(r''_P - r_P)}{(i - Z)^2} + Y^2 \left[\frac{r'_P \cot \theta}{(i - Z)^2} + \frac{r_P + 2r_S - 2(r_P + r_S) \csc^2 \theta}{(i - Z)^2} \right] \right\}, \quad (16)$$

$$B_{z_r} = \theta_0 Y \frac{r_P}{i - Z} + \theta_0^2 XY \left[\frac{r'_P}{(i - Z)^2} - \frac{(r_P + r_S) \cot \theta}{(i - Z)^2} \right]. \quad (17)$$

The expressions above have been calculated for a P -polarized incident beam, and r_P and r_S are the Fresnel reflection amplitude for P and S waves, respectively, evaluated at the central angle of incidence $\theta = \arccos(\mathbf{k}_0 \cdot \hat{\mathbf{z}}/k_0)$. Moreover, we have used the notation $r'_A = \partial r_A / \partial \theta$ and $r''_A = \partial^2 r_A / \partial \theta^2$, where $A \in \{P, S\}$. It is worth noting that the cross-polarization terms at first order in θ_0 present in the expressions of E_{y_r} and B_{x_r} , have the same functional dependence $\sim (r_P + r_S)$ given in Eqs. (16) and (19) of Ref. [6]. Therefore, since for the geometrically reflected beam (which is the specular image of the incident one) we have $r_P = 1$, $r_S = -1$, it is clear that first order cross-polarization terms disappear.

Appendix B

In Eqs. (5-6) u , v , p , q and s are some functions of Z, θ , r_P, r_S and their derivatives. Their explicit form is given below:

$$u = -2r_P r'_P \frac{Z}{1 + Z^2}, \quad (18)$$

$$v = \frac{r_P [r''_P + 2r_S + r'_P \cot \theta - 2(r_P + r_S) \csc^2 \theta]}{2(1 + Z^2)}, \quad (19)$$

$$p = \frac{-r_P(1 - Z^2)(2r''_P - 3r_P) + 2r_P'^2(1 + Z^2)}{4(1 + Z^2)^2}, \quad (20)$$

$$q = \frac{r_P(1 - Z^2) \csc^2 \theta [5r_P + 4r_S + (3r_P + 4r_S) \cos 2\theta - 2r'_P \sin 2\theta]}{2(1 + Z^2)^2}, \quad (21)$$

$$s = \frac{(r_P + r_S)^2 \cot^2 \theta}{1 + Z^2}. \quad (22)$$

These functions look quite complicated, however u , v and q are proportional to r_P so that they disappear when evaluated at Brewster angle of incidence. In passing, we mention here that the linear term $\theta_0 u X$ in Eq. (5) generates the Goos-Hänchen shift in the reflected beam.

References

1. D. Brewster. *Philos. Trans. R. Soc. London*, 105:125, 1815.
2. Y. Fainman and J. Shamir. *Appl. Opt.*, 23:3188, 1984.
3. A. Kőházi-Kis. *Opt. Commun.*, 253:28, 2005.
4. Q. Li and R. J. Vernon. *IEEE Trans. on Antennas and Propagation*, 54:3449, 2006.
5. W. Nasalski. *Opt. Commun.*, 197:217, 2001.
6. W. Nasalski and Y. Pagani. *J. Opt. A: Pure Appl. Opt.*, 8:21, 2006.
7. K. Yu. Bliokh and Yu. P. Bliokh. *Phys. Rev. Lett.*, 96:073903, 2006.

8. A. Aiello and J. P. Woerdman. *Opt. Lett.*, 33:1437, 2008.
9. Throughout this Letter we use the symbols “ \cdot ” and “ \times ”, to denote the ordinary *scalar* and *vector* products in \mathbb{R}^3 , respectively.
10. L. Mandel and E. Wolf. *Optical coherence and quantum optics*. Cambridge University Press, Cambridge, UK, 1 edition, 1995.
11. M. Born and E. Wolf. *Principles of optics*. Cambridge University Press, Cambridge, UK, 7 edition, 2003.
12. R. F. Gragg. *Am. J. Phys.*, 56(12):1092, 1988.
13. M. Lax, W. H. Louisell, and W. B. McKnight. *Phys. Rev. A*, 11:1365, 1975.
14. I. H. Deutsch and J. C. Garrison. *Phys. Rev. A*, 43(3):2498, 1991.
15. In passing, we mention here that the linear term $\theta_0 u X$ in Eq. (5) generates the Goos-Hänchen shift in the reflected beam, where $u = -\frac{2Zr_P}{1+Z^2} \frac{\partial r_P}{\partial \theta}$ for any angle of incidence.
16. R. Simon, E. C. G. Sudarshan, and N. Mukunda. *Appl. Opt.*, 26:1589, 1987.
17. W. L. Erikson and Surendra Singh. *Phys. Rev. E*, 49(6):5778, 1994.